Journal of Aircraft
Vol. 44, No. 3, May–June 2007

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Interval Analysis-Based Optimum Design of Wing Structures Under Taxiing Loads

Luna Majumder* and S. S. Rao†
University of Miami, Coral Gables, Florida 33124-0624

DOI: 10.2514/1.28088

Introduction

An efficient method for including the effect of uncertainty present in the design of aircraft wing structures is presented in this work. By representing each uncertain input parameter as an interval number, the dynamic stresses induced in the wing as the airplane accelerates and decelerates on the runway during takeoff and landing are computed by considering the interaction between the landing gear and the flexible airplane structure. To obtain a physical insight into the nature of the optimum solution an illustrative example is considered and interval-based nonlinear programming techniques are used to find the optimum solution.

Taxiing Stress Analysis

Idealization

The airplane landing gear system is idealized as shown in Fig. 1 for the purpose of taxiing analysis. The airframe is considered as a flexible system. The landing gear system is considered to be the standard oleo-pneumatic shock strut type [1]. The tire is represented as a linear spring. The forces due to the hydraulic resistance of the orifice, air compression, and the internal friction in the shock strut are assumed to be nonlinear. The landing gear forces are assumed to be acting at the landing gear attachment points O_1 and O_2. The aerodynamic lift forces L_1 and L_2 are assumed to pass through their respective aerodynamic centers O_3 and O_4. A set of reference coordinates (x, y, z) is used as shown in Fig. 1, with the origin located at the center of gravity of the airframe. Thus the total vertical displacement of any point on the airframe from the horizontal reference plane can be expressed as

$$w(x,y,t) = \sum_{j=3}^{N} w^j(x,y)\xi_j(t) \tag{1}$$

where $w^{(3)}(x,y)$ is the rigid body translation and $w^{(4)}(x,y)$ is the pitch (rotation about the y axis) of the airframe, $w^{(5)}(x,y),\ldots, w^{(N)}(x,y)$ denote the first $(N-4)$ flexural modes of the airframe, and $\xi_j(t)$ represents the generalized coordinate corresponding to the mode $w^{(j)}(x,y)$, $j = 3,4,\ldots,N$. $\xi_1(t)$ and $\xi_2(t)$ are used to denote the vertical displacements of the unsprung masses M_1 and M_2, respectively. The equations of motion of the airplane can be expressed as [2,3]

$$M_3\ddot{\xi}_3(t) = Q_3(t) \tag{2}$$

$$M_4\ddot{\xi}_4(t) = Q_4(t) \tag{3}$$

$$M_j\ddot{\xi}_j(t) + 2\lambda_j\omega_j M_j\dot{\xi}_j(t) + \omega_j^2 M_j\xi_j(t) = P_j(t); \qquad j = 5,6,\ldots,N \tag{4}$$

where M_j is the generalized mass in the jth mode (M_3 is the gross mass of the airplane and M_4 is the rotary moment of inertia of the airplane about its center of gravity), ω_j is the natural frequency of vibration in the jth mode ($\omega_3 = \omega_4 = 0$), and λ_j is the structural damping coefficient in the jth mode of the airframe. $P_j(t)$ is the generalized forcing function of the jth mode given by

$$P_j(t) = -(Q_1 - \bar{Q}_1)w^j(x_1,y_1) - (Q_2 - \bar{Q}_2)w^j(x_2,y_2) - L_1 w^j(x_3,y_3) - L_1 w^j(x_4,y_4) \tag{5}$$

Here Q_1 and Q_2, the total interacting forces between the airframe and the main and nose landing gears passing through the points O_1 and O_2, respectively, are given by the sum of the hydraulic resistance of the orifice (F_{di}), the air compression or pneumatic force (F_{si}) and the internal friction (F_i) in the suspension system. $\bar{Q}_1$ and $\bar{Q}_2$ represent the corresponding static values when the speed of the airplane is zero. $w^{(j)}(x_k, y_k)$ is the value of the mode shape $w^{(j)}(x,y)$ at point O_k. $k = 1$ and $k = 2$ correspond to the points O_1 and O_2 and $k = 3$ and $k = 4$ refer to the aerodynamic centers O_3 and O_4 in Fig. 1. The equation of motion of the unsprung mass M_i is established by considering the mass itself as a free body:

$$M_i\ddot{\xi}_i = W_i + P_i - F_{ti}, \qquad i = 1,2 \tag{6}$$

where $P_i = F_{di} + F_{si} + F_i$, and F_{ti} is the interacting force between the wheel of the ith landing gear and the tire force.

Solution of the Equations of Motion

The N equations of motion of the unsprung masses, Eqs. (2–4) and (6), form a set of nonlinear differential equations of motion in terms of N unknowns $\xi_i(t)$, $i = 1,2,\ldots,N$. These equations are solved numerically by means of a step by step method of integration. For this, the time required by the airplane to traverse a certain distance of runway is divided into a number of short intervals Δt. The linear acceleration method [4,5] is used to find the values of the generalized displacement $\xi_i(t)$, velocity $\dot{\xi}_i(t)$, and acceleration $\ddot{\xi}_i(t)$ at time t from the known values of $\xi_i(t)$, $\dot{\xi}_i(t)$, and $\ddot{\xi}_i(t)$ at time $t - \Delta t$. For a simplified analysis, the nonlinear time-dependent damping force and spring force in the suspension system are linearized [3] and the equations of motion (2–4) and (6) can be written in matrix form as

Received 15 November 2006; revision received 22 January 2007; accepted for publication 6 February 2007.

*Graduate Student, Department of Mechanical and Aerospace Engineering.
†Professor and Chairman, Department of Mechanical and Aerospace Engineering; srao@miami.edu (Corresponding Author).

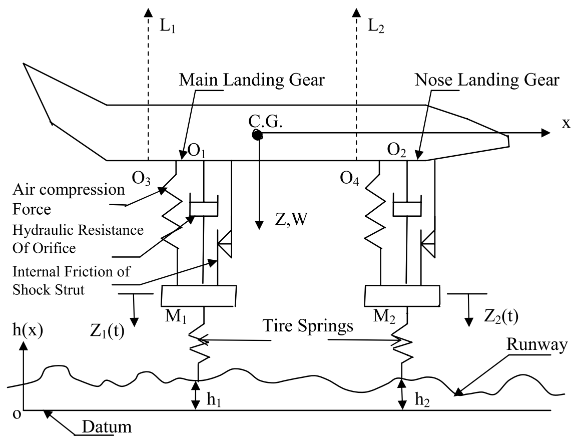

Fig. 1 Idealization of the aircraft for taxiing analysis.

$$[M]\{\ddot{\xi}\} + [C]\{\dot{\xi}\} + [K]\{\xi\} = \{R\}_t \tag{7}$$

where $[M]$, $[C]$, and $[K]$ denote mass, damping, and stiffness matrices, respectively, $\{R\}_i$ represents the forcing function vector of the system, and $\{\xi\}$ indicates the vector of generalized displacements of the airframe landing gear system. It is to be noted that before considering the explicit form of the elements of $[M]$, $[C]$, $[K]$, and $\{R\}_t$, four possible situations have to be distinguished because of the presence of the Coulomb friction in the landing gear system [1].

Optimization Problem

The optimization of a supersonic wing is considered for illustration. The structural weight of the wing shown in Fig. 2 is minimized. The thicknesses of the skin, the thickness of the ribs and spars, and the cross-sectional areas of the pin-jointed bars are treated as design variables. The taxiing stress is constrained by an upper bound and the design variables are restricted to lie between specified limits. The optimization problem can be stated as as follows:

Find $\boldsymbol{X} = \{x_1, x_2, \ldots, x_6\}$ which minimizes

$$f(\boldsymbol{X}) = \sum_{i=1}^{N_s} \rho t_{si} A_{si} + \sum_{i=1}^{N_w} \rho t_{wi} A_{wi} + \sum_{i=1}^{N_b} \rho l_{bi} A_{bi} \tag{8}$$

subject to constraints on other response quantities that include wing tip deflection, root angle of attack, steady-state stress, and bending and torsional natural frequencies as well as the design variables.

Because all the system parameters, including the design variables and the objective function, are treated as interval variables, we need

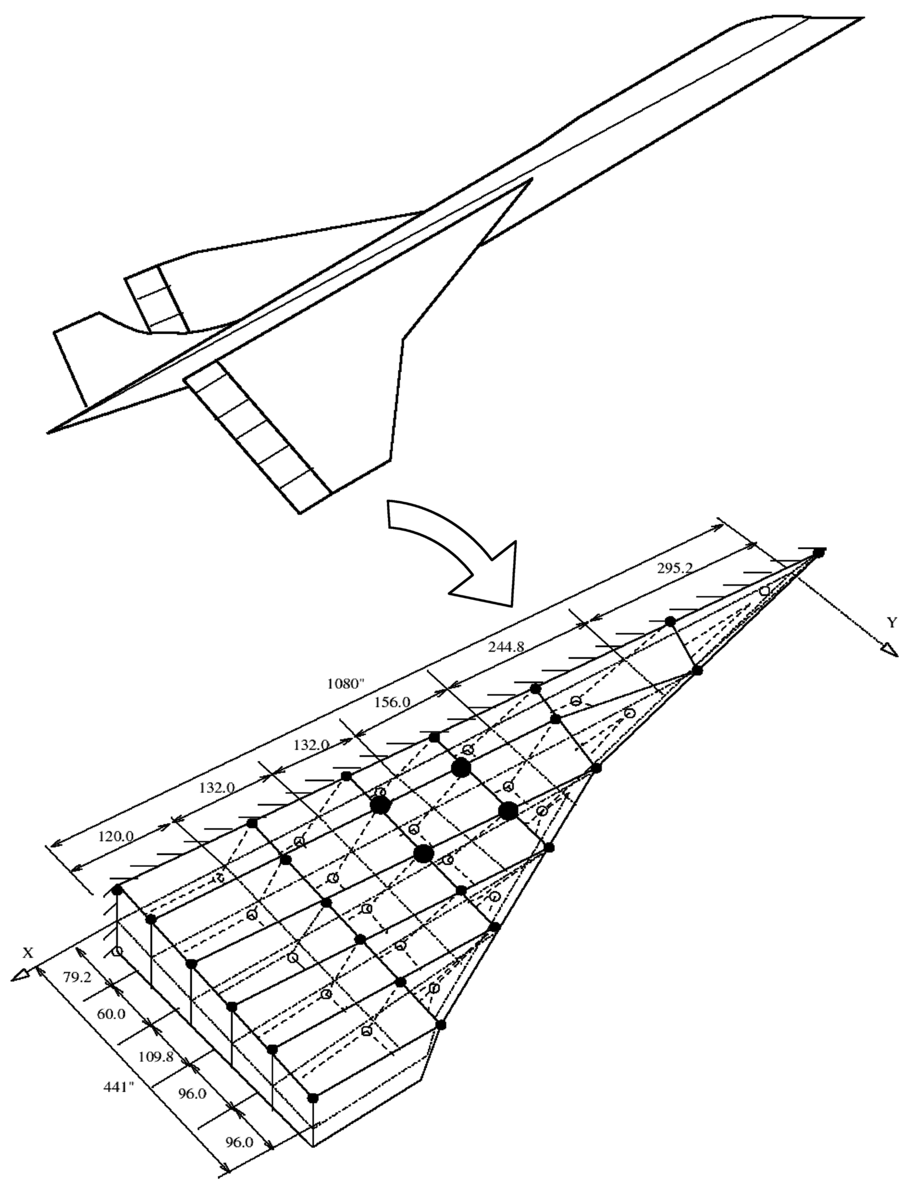

Fig. 2 Supersonic transport wing, finite element idealization.

Table 1 Design data for example wing

Material properties:	Material	Alumium
	Young's modulus	10×10^6 psi
	Poisson's ratio	0.333
	Density	$0.10\ \mathrm{lb/in.}^3$
Details of the weight:	Planform area	$1474.5\ \mathrm{ft}^2$
	Engines	12,500 lb
	Fuselage and payload	72,000 lb
	Fuel	92,500 lb
	Initial gross weight	192,500 lb
Flight conditions data:		
Altitude		25,000 ft
Pull-up acceleration		3.75 g
Flight Mach number		1.89
Pressure of air		$786.33\ \mathrm{lb/ft}^2$
Density of air		$0.001066\ \mathrm{lb \cdot s^2/ft^4}$
Taxiing analysis data:		
$N = 5$, $\Delta t = 0.002$ s, $\lambda_j = 0.025$, $M_{1g} = 4992.0$ lb, $M_{2g} = 342.0$ lb, $M_{3g} = 386,464$ lb, $M_4 = 0.645 \times 10^8\ \mathrm{lb \cdot in. \cdot s^2}$, $k_{t1} = 96,500.0$ lb/in., $k_{t2} = 13,500$ lb/in., $F'_1 = 1000$ lb, $F'_2 = 600$ lb, $F_{s1} = 33,000.0$ lb, $F_{s2} = 3500.0$ lb		
Optimization data:		
For design variables: lower bound = 0.04, upper bound = 0.5; upper bounds: taxiing stress (σ_t) = $3.5e4$ lb/in.^{2}, wing tip deflection (δ) = 60 in., steady-state stress (σ_s) = $5.5e4$ lb/in.^{2}, root angle of attack (α_0) = 12 deg; lower and upper bounds: bending frequency = 3.0 rad/s and 10.0 rad/s; torsional frequency = 10.0 rad/s and 25 rad/s; lower bound: flutter Mach number = 1.89.		

to apply interval arithmetic to every step of the calculations. During actual programming, we need to adjust the order in which different interval parameters are considered in any specific equation. This is because, when the program executes the equation using interval parameters, the new order will not only minimize the computational time but also lead to a reduced interval range for the result. In addition, the truncation approach [6] is used based on a comparison between the ranges of the input parameters and the range of the computed response. The purpose of truncation is to make reasonable modifications to the output range before applying it to the next interval operation. This truncation approach can give reasonable predictions for the solution even when the widths of their starting points or the interval ranges of other influencing parameters are quite large.

In some computational steps, using interval arithmetic may not seem only to be redundant, but might lead to an invalid result which does not follow the physics of the equation. If these invalid operations are used in the computation, the final solution will be incorrect. In such cases, it is safe to apply a combinatorial approach instead of the interval operation to comply with the physical logic. Thus it is necessary to understand the physical meaning of each equation before implementing the interval analysis. For interval analysis, each input parameter described in Table 1 is represented as a range or interval as $(x - \Delta x, x + \Delta x)$, where x is the mean or deterministic value of the parameter and Δx is the deviation from the mean value, taken as $0.05x$. For comparison purposes, a probabilistic analysis is performed by representing each parameter as a random variable following normal distribution with known mean value x and standard deviation σ_x which is taken as one-third of Δx. By computing the mean value and standard deviations of outputs from the mean value and standard deviations of the uncertain input parameters, the constraints for the probabilistic optimization are stated as

$$\bar{g} + 3S_g \leq g_{\mathrm{limit}} \tag{9}$$

where $\bar{g}$ is the mean value, S_g is the standard deviation, and g_{limit} is the allowable limit for each constraint, which is taken as deterministic for simplicity.

Solution Procedure

The multivariable constrained optimization problems are solved using nonlinear programming techniques (penalty function approach and sequential quadratic programming).

Table 2 Comparison of optimization results obtained with different approaches

Design parameters	Deterministic analysis	Probabilistic analysis	Interval analysis
Design variables,			
x_1 in.	0.0923	0.1109	[0.0895 0.0946]
x_2 in.	0.0513	0.0412	[0.0502 0.0550]
x_3 in.	0.0421	0.0519	[0.0419 0.0426]
x_4 in.	0.0407	0.0426	[0.0403 0.0411]
x_5 in.	0.1647	0.1572	[0.1566 0.1839]
x_6 in.^{2}	0.0544	0.0406	[0.0525 0.0564]
Wing tip deflection, δ, in	49.1517[a]	49.98[a]	[48.880 57.036][a]
Root angle of attack, α_0 rad (deg)	0.1530	0.1525	[0.1523 0.1538]
	(8.7656)	(8.7408)	[8.7241 8.8139]
First natural frequency, ω_1, rad/s	6.3119	5.2272	[4.1286 6.9808]
Second natural frequency, ω_2, rad/s	14.2102	14.3591	[13.687 15.053]
Flutter Mach no. M_F	3.1506	2.9845	[2.9513 3.7213]
Steady-state stress, σ_s, lb/in.^{2}	$4995e4$[a]	$4.2251e4$	[$4.7715e4$ $5.353e4$][a]
Taxiing stress, σ_t, lb/in.^{2}	$2.9817e4$[a]	$2.9498e4$[a]	[$2.311e4$ $3.306e4$][a]
Structural weight of wing, (objective function), lb	7340.348	7976.686	[7717.7 7507.2]

[a]Active constraint.

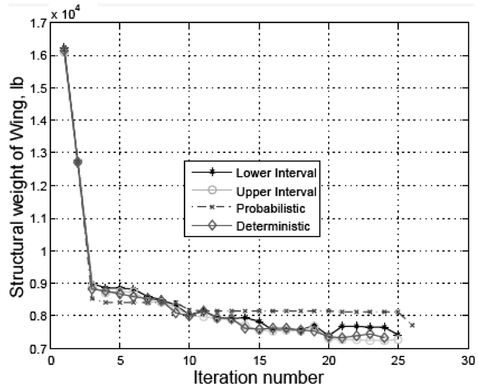

Fig. 3 Optimization of example wing.

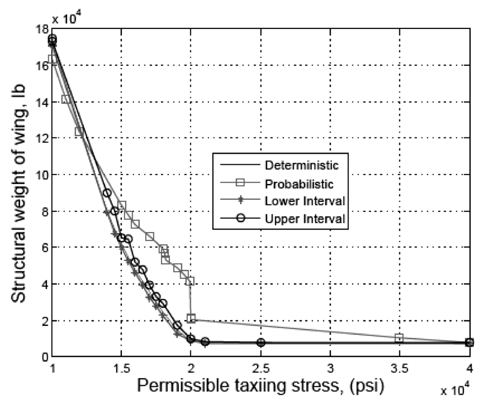

Fig. 4 Sensitivity of structural weight to taxiing stress.

Numerical Results and Discussion

The pertinent data for the example wing structure shown in Fig. 2 are given in Table 1. The finite element method is used for the modeling of the wing structure. Because there are a large number of interval operations in each expression, and a specific interval may appear several times in different terms in the same equation, the widths or ranges of stresses are wider than the true widths. To avoid this unrealistic growth of intervals, the truncation method [6] has been used. A computer program is developed for finding the optimum solutions. For comparison, the problem is also solved by considering all the constraints to be probabilistic. For the interval analysis the problem is solved by treating all the design parameters as interval numbers. The results of optimization obtained with deterministic, probabilistic, and interval analyses are reported in Table 2. Figure 3 shows the convergence of objective functions with the number of iterations, for deterministic, probabilistic, and interval optimizations. Figure 4 shows the sensitivity of structural weight with the maximum permissible value of the taxiing stress.

Conclusions

The feasibility of performing the automated optimum design of airplane wing structures at preliminary design stages, with consideration of the taxiing stress developed, is studied. The results obtained with the interval approach are compared to those obtained with deterministic and probabilistic analyses. The optimization results obtained with the interval analysis are found to be in good agreement with those obtained with the deterministic approach. The interval analysis is expected to be more realistic and, hence, should be used in the optimum design of the airplane wing structure.

References

[1] Milwitzky, B., and Cook, F. E., "Analysis of Landing Gear Behavior," NACA TR 1154, 1953.

[2] Rao, S. S., "Optimization of Airplane Wing Structures Under Taxiing Loads," *Computers and Structures*, Vol. 26, No. 3, 1987, pp. 469–479.

[3] Tung, C. C., Penzien, J., and Horonjeff, R., "The Effect of Runway Unevenness on the Dynamic Response of Supersonic Transports," NACA CR-119, 1964.

[4] Timoshenko, S. P., Young, D. H., and Weaver, W., Jr., "Vibration Problems in Engineering," 4th ed., Wiley, New York, 1974.

[5] Rao, S. S., "Mechanical Vibrations," Addison–Wiley, Reading, MA, 1986.

[6] Rao, S. S., and Berke, L., "Analysis of Uncertain Structural Systems Using Interval Analysis," *AIAA Journal*, Vol. 35, No. 4, 1997, pp. 727–735.